

Cascaded noiseless linear amplification for single-photon state

Lan Zhou^{1,2}, Yu-Bo Sheng^{2,3,*}

¹ College of Mathematics & Physics,
Nanjing University of Posts and Telecommunications,
Nanjing, 210003, China

² Key Lab of Broadband Wireless Communication and Sensor Network Technology,
Nanjing University of Posts and Telecommunications,
Ministry of Education, Nanjing, 210003, China

³ Institute of Signal Processing Transmission,
Nanjing University of Posts and Telecommunications,
Nanjing, 210003, China

(Dated: November 5, 2014)

Photon loss is one of the main obstacles in current long-distance quantum communications. The approach of noiseless linear amplification (NLA) is one of the powerful way to distill the single-photon state (SPS) from a mixed state, which comprises both the SPS and vacuum state. However, existing NLA protocol can only perform the amplification for one time. That is the fidelity of the SPS cannot be increased anymore. In this paper, We put forward an efficient cascaded NLA protocol for both the SPS and single-photon entanglement (SPE), respectively, with the help of some auxiliary single photons. By repeating this protocol for sever times, the fidelity of the SPS and SPE can reach near 100%, which may make this protocol is extremely useful to close the detection loophole in quantum key distribution. Moreover, this protocol is based on the linear optics, which makes it feasible in current technology.

PACS numbers: 03.67.Mn, 03.67.Hk, 03.65.Lx

I. INTRODUCTION

In the past few decades, a series of attractive achievements in the quantum information field have been made. Quantum computation and quantum communication will no longer be a dream [1]. In quantum communication, there are some important protocols have been proposed, such as quantum teleportation [2], quantum key distribution [3, 4], quantum state sharing [5], quantum secure direct communication [6], and so on [7–11]. Photon is the best candidate to carry and distribute quantum information, because it has fast transmission speed and is easy to control. Unfortunately, the unavoidable absorption and scattering in a transmission quantum channel places a serious limitation on the length of the communication distances [12]. The photon loss becomes one of the main obstacles in long-distance quantum communication. It not only decreases the efficiency of the communication, but also will make the communication insecure, because the detection loophole [13–15].

During the past decades, people developed two powerful quantum technologies to resist the photon loss. The first quantum technology is the quantum repeaters [12, 16, 17]. By dividing the whole channel into several segments, they first generate the entanglement in each segment. Finally, by entanglement swapping, they can set up the entanglement in the whole distance. The second quantum technology which will be detailed in this paper is the quantum state amplification. Though the quantum repeater can extend the entanglement between the adjacent segments, they still require to distribute entanglement in each short-distance segment. In this way, during the transmission, the photons will lose with some probability. Briefly speaking, the photon loss will cause the single photon $|1\rangle$ degrade to a mixed state as $\eta|1\rangle\langle 1| + (1 - \eta)|0\rangle\langle 0|$, which means the photon may be completely lost in the probability of $1 - \eta$.

In 2009, Ralph and Lund first proposed the concept of the noiseless linear amplification (NLA) to distill the new mixed state with relatively high fidelity from the input mixed state with low fidelity [18]. Since then, various NLA protocols have been proposed [19–31]. Current NLA protocol can be divided into three groups. The first group focused on the single photon [19–22]. The second group focused on the single-photon entanglement (SPE) [23–28], for the SPE is the simplest entanglement form, but it has important applications in cryptography [32, 33], state engineering [34], tomography [35, 36], entanglement purification [37, 38], entanglement concentration [39–43]. The third group focused on the continuous variables systems [29–31]. For example, in 2012, Osorio *et al.* experimentally realized the heralded noiseless amplification for the single-photon state (SPS) with the help of the single-photon source and the linear optics [19]. Kocsis *et al.* demonstrated the heralded noiseless amplification of a photon polarization qubit [20]. Zhang *et al.* also proposed an NLA protocol for protecting the SPE [25].

So far, all the existing NLA protocols for the SPS and SPE can only be performed for one time. That is the fidelity of the initial state can be increased for one step. In the practical high noisy applications, the photon loss is usually

high. In this way, after performing the amplification protocol, the quality of the entanglement may not reach the standard for secure and highly efficient long-distance quantum communication. Because in order to close the detection loophole, the fidelity of the state is the higher the better. In this way, we should seek for the efficient approach to realize the amplification. In this paper, based on the linear optics, we propose an efficient NLA protocol for protecting both the SPS and SPE, respectively. In our protocol, the amplification is cascaded. That is the fidelity of the SPS and SPE can be increased step by step.

This paper is organized as follows: In Sec. II, we present the cascaded NLA protocol for SPS. In Sec. III, we present our cascaded amplification protocol for SPE. In Sec. IV, we present a discussion. Finally, in Sec. V, we present a conclusion.

II. CASCADED NLA PROTOCOL FOR SPS

Before we start to describe our protocol, we first introduce the basic principle of the NLA protocol, based on the work of Gisin *et al.*[19]. It is composed of the variable fiber beam splitter (VBS) with the transmittance of t and the 50:50 beam splitter (BS). The schematic drawing is shown in Fig. 1. The mixed state of the SPS can be described as

$$\rho_{in} = \eta|1\rangle_{a1}\langle 1| + (1 - \eta)|0\rangle_{a1}\langle 0|. \quad (1)$$

In Ref.[19], an auxiliary single photon is required. The auxiliary photon passes through the VBS, which can generate a single-photon entangled state as

$$|\varphi_A\rangle = \sqrt{t}|1\rangle_{b1}|0\rangle_{b2} + \sqrt{1-t}|0\rangle_{b1}|1\rangle_{b2}. \quad (2)$$

We make the photons in the a1 and b1 modes pass through the BS, which can make

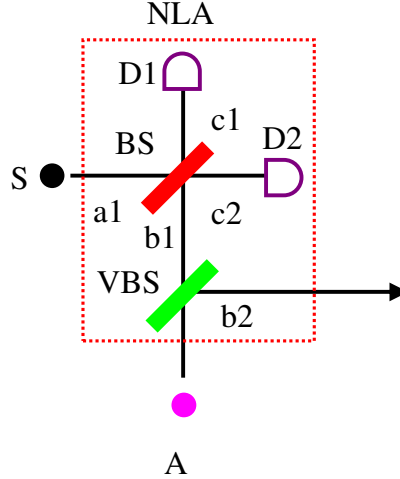


FIG. 1: The schematic drawing of the NLA protocol in Ref. [19] for SPS. S represents the single-photon source. VBS means the variable fiber beam splitter, and BS means the 50:50 beam splitter. By passing through the NLA unit, the output mixed state with higher fidelity can be distilled.

$$|1\rangle_{a1} = \frac{1}{\sqrt{2}}(|1\rangle_{c1} + |1\rangle_{c2}), \quad |1\rangle_{b1} = \frac{1}{\sqrt{2}}(|1\rangle_{c1} - |1\rangle_{c2}). \quad (3)$$

Then, the output photon in the c1 and c2 modes are detected by the single-photon detectors D1 and D2, respectively. We select the cases which make D1 or D2 detects exactly one photon. In this way, we can finally obtain a new mixed state as

$$\rho_{out1} = \eta_1|1\rangle_{b2}\langle 1| + (1 - \eta_1)|0\rangle_{b2}\langle 0|, \quad (4)$$

where the coefficient η_1 is

$$\eta_1 = \frac{(1-t)\eta}{(1-t)\eta + (1-\eta)t} = \frac{(1-t)\eta}{\eta - 2\eta t + t}. \quad (5)$$

The success probability of the protocol is

$$P = (1 - t)\eta + t(1 - \eta) = t + \eta - 2t\eta. \quad (6)$$

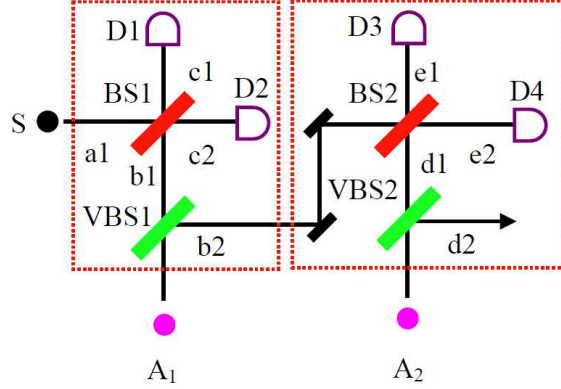


FIG. 2: The schematic drawing of the two-level NLA protocol for SPS.

We denote the amplification factor $g \equiv \frac{\eta_1}{\eta}$, so that we can obtain

$$g = \frac{1 - t}{\eta - 2\eta t + t}. \quad (7)$$

In order to realize the amplification, it is required that $g > 1$. It can be calculated that we can obtain $g > 1$ only if $t < \frac{1}{2}$.

In the section, we put forward an efficient cascaded amplification protocol for the SPS with the help of the NLA unit. We first describe the two-level cascaded NLA protocol, as described in Fig. 2. From Fig.2, we require two single photons A_1 and A_2 as auxiliary. After one of the single-photon detectors $D1$ or $D2$ registers one photon, the state in the spatial mode $b2$ must be an amplified mixed state of the form in Eq.(4). In this way, the new mixed state can be regarded as the initial state in the second amplification, with the help of another auxiliary single photon A_2 . In the second round, we still choose the case that only one of the single-photon detectors $D3$ or $D4$ register the photon. If the protocol is successful, we can obtain a new mixed state as

$$\rho_{out2} = \eta_2|1\rangle_{d2}\langle 1| + (1 - \eta_2)|0\rangle_{d2}\langle 0|, \quad (8)$$

with

$$\eta_2 = \frac{\eta_1(1 - t)}{t + \eta_1 - 2\eta_1 t} = \frac{\eta(1 - t)^2}{t^2 + \eta - 2\eta t}. \quad (9)$$

It is straightforward to extend this two-level cascaded NLA protocol to the arbitrary level cascaded NLA protocol, as described in Fig. 3. In the protocol, we make the input photon pass through N NLA units, successively. The output photon state from the previous NLA unit enter the next NLA unit as the input one. The transmittance of the VBS in each NLA unit is required to meet $t < \frac{1}{2}$. With the help of N auxiliary photons, say A_1, A_2, \dots, A_N , the initial SPS can be cascaded to amplify for N time. Based on the description in Sec. II, after N NLA unit, we can obtain the output new mixed photon state as

$$\rho_{outN} = \eta_N|1\rangle\langle 1| + (1 - \eta_N)|0\rangle\langle 0|. \quad (10)$$

The η_N is the fidelity of the output mixed state after NLA_N . We can calculate the fidelity of the mixed state after the photon pass through each NLA unit as

$$\begin{aligned} \eta_1 &= \frac{\eta(1 - t)}{t + \eta - 2\eta t}, \\ \eta_2 &= \frac{\eta_1(1 - t)}{t + \eta_1 - 2\eta_1 t}, \\ &\dots\dots\dots \\ \eta_N &= \frac{\eta_{N-1}(1 - t)}{t + \eta_{N-1} - 2\eta_{N-1} t}. \end{aligned} \quad (11)$$

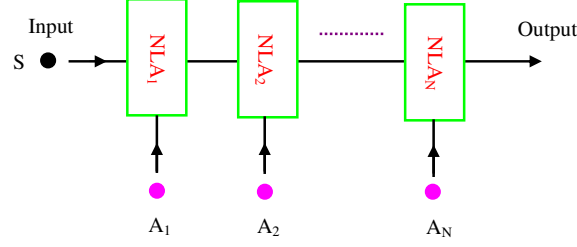


FIG. 3: The schematic drawing of our cascaded amplification protocol for SPS. We make the input photon state pass through N NLA units, successively. With the help of N auxiliary single photons, we can realize the cascade amplification for the SPS.

After NLA_N , we define the total amplification factor G as

$$G \equiv \frac{\eta_N}{\eta} = \frac{\eta_N}{\eta_{N-1}} \frac{\eta_{N-1}}{\eta_{N-2}} \cdots \frac{\eta_2}{\eta_1} \frac{\eta_1}{\eta} = g_1 g_2 \cdots g_N. \quad (12)$$

Under the case that the transmittance of each VBS meet $t < \frac{1}{2}$, we can ensure arbitrary $g_n > 1$. Therefore, increasing the number of NLA unit (N) can effectively increase the fidelity of the final output SPS. Especially, if $N \rightarrow \infty$, we can make $\eta_N \rightarrow 1$.

Meanwhile, the success probability to distill the new mixed state after each NLA can be written as

$$\begin{aligned} P_1 &= t + \eta - 2t\eta, \\ P_2 &= P_1[t + \eta_1 - 2\eta_1 t], \\ &\dots\dots\dots \\ P_N &= P_{N-1}[t + \eta_{N-1} - 2\eta_{N-1} t]. \end{aligned} \quad (13)$$

Obviously, as arbitrary $P_n < 1$, increasing the number of the NLA unit will reduce the success probability. Therefore, it is a trade-off between the fidelity and the success probability. In order to obtain the SPS with high fidelity, we need to consume large number of the input single photons.

III. CASCADE AMPLIFICATION FOR THE SPE

In 2012, the setup of NLA described in Ref. [19] was developed to amplify SPE [20], which is shown in Fig. 2. Briefly speaking, a single-photon source S emits single-photon entangled state, which is in the spatial mode $a1$ and $b1$. The form of SPE can be written as

$$|\phi\rangle_{AB} = \frac{1}{\sqrt{2}}(|10\rangle_{AB} + |01\rangle_{AB}). \quad (14)$$

Due to the photon loss, the SPE degrades to a mixed state as

$$\rho_{in} = \eta' |\phi\rangle_{AB} \langle \phi| + (1 - \eta') |vac\rangle \langle vac|. \quad (15)$$

For realizing the amplification, two auxiliary photons are required and both the two parties need to run the same operation as described above, simultaneously. After the amplification, we can obtain the new mixed state as

$$\rho_{out2} = \eta'_1 |\phi\rangle_{AB} \langle \phi| + (1 - \eta'_1) |vac\rangle \langle vac|, \quad (16)$$

with the success probability of

$$P' = \eta' t (1 - t) + (1 - \eta') t^2 = (1 - 2\eta') t^2 + \eta' t. \quad (17)$$

The fidelity of the new mixed state is

$$\eta'_1 = \frac{\eta' (1 - t)}{\eta' (1 - t) + (1 - \eta') t} = \frac{\eta' (1 - t)}{\eta' - 2\eta' t + t}. \quad (18)$$

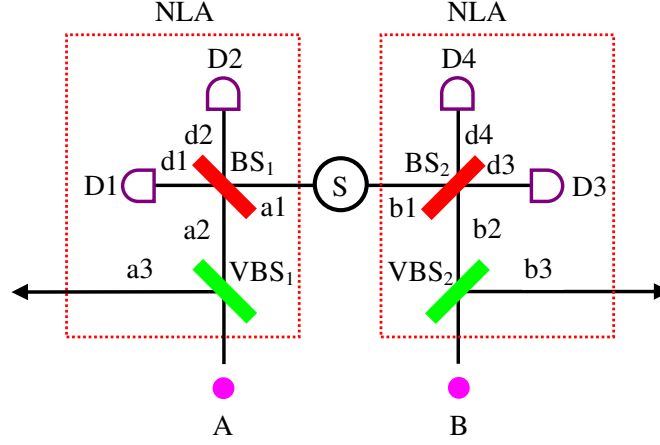


FIG. 4: The schematic drawing of the NLA protocol in Ref. [25] for the SPE. In the protocol, both the two parties makes the photons in their hands pass through a NLA unit, simultaneously. With the help of two auxiliary single photons, we can distill the new mixed state with higher fidelity.

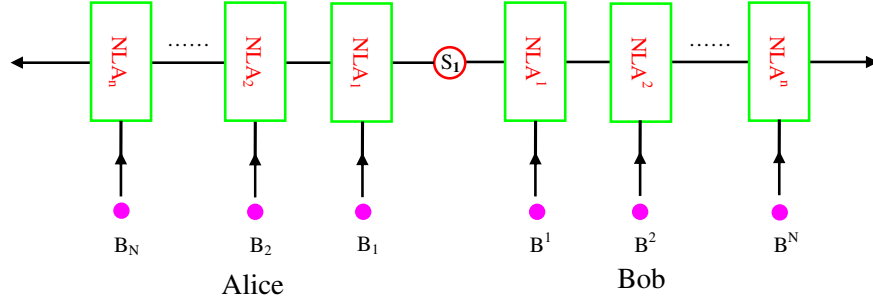


FIG. 5: The schematic drawing of our cascaded amplification protocol for SPE. Each of the two parties makes the photon in his/her hand pass through N NLA units, successively. With the help of 2N auxiliary single photons, we can realize the cascade amplification for the single-photon entangled state.

Interestingly, it can be found that the form of the fidelity in Eq. (18) is the same as that in Eq. (5). Therefore, we can also obtain that when the transmittance of each VBS meets $t < \frac{1}{2}$, the amplification factor $g > 1$.

In the section, the NLA unit will be used to realize the cascade amplification for the SPE state. As shown in Fig. 5, due to the environmental noise, Alice and Bob share a mixed state as Eq. (15). For realizing the amplification, each of them needs to prepare N NLA units, say $NLA_1, NLA_2, \dots, NLA_N$, and $NLA^1, NLA^2, \dots, NLA^N$, respectively. Each of Alice and Bob makes the single photon in his/her hand pass through the N units, simultaneously. Similarly, during the amplification process, each of them needs to introduce N auxiliary photons, say B_1, B_2, \dots, B_N , and B^1, B^2, \dots, B^N . After the N NLA units, we can obtain the final output quantum state as

$$\rho'_{out} = \eta'_N |\phi\rangle_{AB} \langle \phi|_{AB} + (1 - \eta'_N) |vac\rangle \langle vac|, \quad (19)$$

where η'_N is the fidelity of the mixed state when both the input photons from Alice and Bob pass through N NLA units.

We can also calculate the fidelity and success possibility of the cascaded amplification protocol for the SPE state. The fidelity of the mixed state when both the two parties make the input photon pass through arbitrary N NLA units can be written as

$$\begin{aligned} \eta'_1 &= \frac{\eta'(1-t)}{\eta'(1-t) + (1-\eta')t}, \\ \eta'_2 &= \frac{\eta'_1(1-t)}{\eta'_1(1-t) + (1-\eta'_1)t}, \end{aligned}$$

$$\dots\dots\dots \eta'_N = \frac{\eta'_{N-1}(1-t)}{\eta'_{N-1}(1-t) + (1-\eta'_{N-1})t}, \quad (20)$$

where the subscript "1", "2", $\dots\dots\dots$ "N" mean the number of the NLA units adopted by each of the two parties. Similarly, it can be found Eq. (20) has the same form as Eq. (11).

Therefore, when both the two parties make their photon pass through N NLA units, the total amplification factor G is the same as Eq. (12). In this way, under the case that $t < \frac{1}{2}$, we can effectively increase the amplification factor by increasing the number of the NLA units.

Certainly, we can also calculate the success probability of the cascaded amplification protocol as

$$\begin{aligned} P'_1 &= (1 - 2\eta')t^2 + \eta't, \\ P'_2 &= P'_1[(1 - 2\eta'_1)t^2 + \eta'_1 t], \\ &\dots\dots\dots \\ P'_N &= P'_{N-1}[(1 - 2\eta'_{N-1})t^2 + \eta'_{N-1} t], \end{aligned} \quad (21)$$

where the subscript "1", "2", $\dots\dots\dots$ "N" are the number of the NLA units used in each of the two parties. Similarly, increasing the fidelity will also sacrifice the success probability. For obtaining the SPE with high fidelity, we still need to consume large amount of initial input state.

IV. DISCUSSION

So far, based on the work of Refs.[19, 25], we have fully described our cascaded NLA protocol for both SPS and SPE. The NLA unit which is composed of the VBS and BS, is the key element of the two protocols. In our protocol, we make the target photon pass through N NLA units, successively. Under the case that the transmittance of the VBS (t) meets $t < \frac{1}{2}$, when the target photon pass through a NLA unit, we can realize an amplification with the help of an auxiliary photon. In this way, by making the photon pass through N NLA units successively, we can finally realize N cascaded amplification.

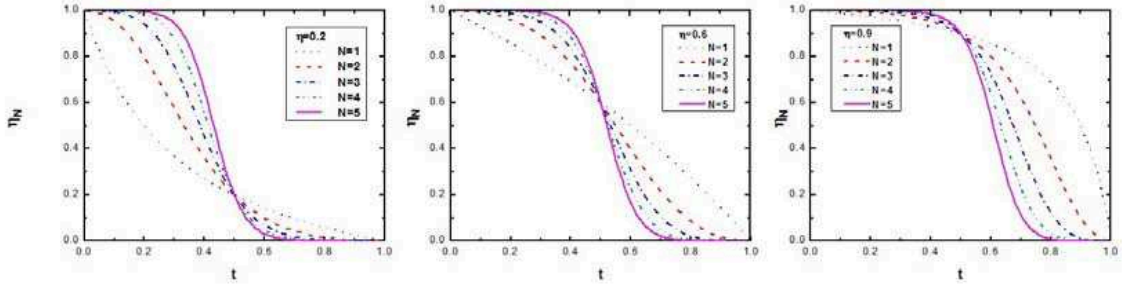


FIG. 6: The fidelity (η_N) of the distilled new mixed state as a function of the transmittance (t) of the VBS, when N (2N) NLA units were used to cascade amplify the SPS (SPE). For comparison, we make N=1, 2, 3, 4 and 5, respectively. Meanwhile, we suppose that both the two protocols are operated under low initial fidelity ($\eta = 0.2$), medium initial fidelity ($\eta = 0.6$), and high initial fidelity ($\eta = 0.9$).

According to Eq. (11) and Eq. (20), the fidelity (η_N) of the output mixed states largely depends on the initial fidelity (η) of the input state, the transmittance (t) of the VBS, and the number (N) of the NLA unit. Fig. 6 shows η_N as a function of t . For comparison, we suppose the protocols are operated under low initial fidelity ($\eta = 0.2$), medium initial fidelity ($\eta = 0.6$) and high initial fidelity ($\eta = 0.9$), respectively, and make N=1, 2, 3, 4, and 5. The fidelity curve of the protocols in Ref. [19, 20] is the same as that for $N = 1$. It can be found that η_N reduces with the growth of t . The five curves in each figure intersect at the point of $t = 0.5$. Under the case that $t = 0.5$, $\eta_N = \eta$, that is, the output mixed state is the same as the input state. Actually, when $t = 0.5$, the VBS become the BS, the whole amplification process is converted to the standard teleportation process. On the other hand, increasing the number of the NLA unit can effectively increase the fidelity of the output states, especially under low initial fidelity condition. In practical applications, the photon loss is usually high. Fig. 7 shows the the fidelity η_N altered with

N under practical high photon loss condition ($\eta = 0.2$). It can be found that when $t = 0.2$, η_1 is only 0.5, while η_5 can reach 0.996. Therefore, by selecting the suitable VBS and the suitable value of N , we can make $\eta_N \rightarrow 1$. In this way, our protocols may provide an effectively way to close the detection loophole in QKD [30]. Certainly, we should point out that though the η_N increases with N , we cannot reach $\eta_N = 1$. The limitation of $\lim_{N \rightarrow \infty} \eta_N = 1$. That is the $\eta_N = 1$ is a fixed point for the iterative equations in both Eq. (11) and Eq. (20). On the other hand, under high photon loss condition ($\eta = 0.2$), the protocols in Refs. [19, 20] can obtain relatively high fidelity only under the extreme condition that $t \rightarrow 0$. For example, when $N = 1$, η_1 is 0.69 under $t = 0.1$, and η_1 can reach 0.96 under $t = 0.01$. In current experimental conditions, the VBS with $t \rightarrow 0$ is unavailable. However, with the growth of N , the requirement for t is largely reduced. For example, when $N = 5$, η_5 can reach 0.996 under $t = 0.2$. Therefore, it is much easier for us to obtain high fidelity under practical experimental conditions.

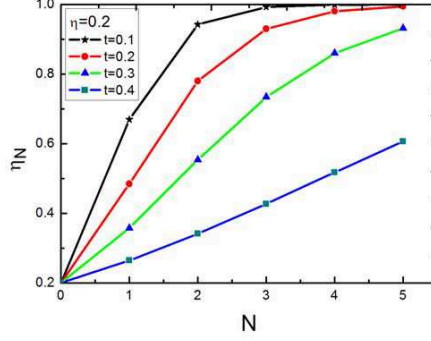


FIG. 7: The fidelity η_N altered with the iteration number N . Here we let $N = 1, 2, 3, 4, 5$ and the initial fidelity $\eta = 0.2$.

Finally, we will discuss the experimental realization of our protocol. The VBS and BS are the key elements. Ref. [19] reported the experimental results about the NLA with the help of the VBS. The protocol can increase the probability η_t of the single photon $|1\rangle$ from a mixed state $\eta_t|1\rangle\langle 1| + (1 - \eta_t)|0\rangle\langle 0|$ by adjusting the splitting ratio of VBS from 50:50 to 90:10. Based on the experiments, our requirement for $t < 1/2$ can be easily realized under current technology. In our protocol, the processing of the photons passing through the BS is essentially the Hong-Ou-Mandel (HOM) interference, so that the two photons should be indistinguishable in every degree of freedom. Ref. [19] also measured the HOM interference on each BS. Their experimental results for each BS are $93.4 \pm 5.9\%$ and $92.1 \pm 5.7\%$, respectively. In Fig. 8, we design the possible experimental realization of the two-level amplification for the SPS. Based on Ref. [44–47], with the help of spontaneous parametric down-conversion (SPDC) source, we make a pump pulse of ultraviolet light pass through a beta barium borate (BBO) crystal and produce correlated pairs of photons into the modes a_1 and b_1 . Then it is reflected on the mirror and traverses the crystal for a second time, and produces correlated pairs of photons into the modes a_2 and b_2 . The Hamiltonian can be approximately described as

$$\begin{aligned}
 H_{PDC} = & |0\rangle + p(|a_1^\dagger b_1^\dagger\rangle + |a_2^\dagger b_2^\dagger\rangle)|0\rangle + p^2(|a_1^\dagger b_1^\dagger\rangle \otimes |a_2^\dagger b_2^\dagger\rangle \\
 & + |a_1^\dagger b_1^\dagger\rangle \otimes |a_1^\dagger b_1^\dagger\rangle + |a_2^\dagger b_2^\dagger\rangle \otimes |a_2^\dagger b_2^\dagger\rangle)|0\rangle + o(p).
 \end{aligned} \tag{22}$$

In the experiment, we select the item $(|a_1^\dagger b_1^\dagger\rangle \otimes |a_2^\dagger b_2^\dagger\rangle)|0\rangle$ with the help of the coincidence measurement, which generates four photons in the modes a_1 , b_1 , c_1 , and d_1 , simultaneously. Under this case, we make the photon in a_1 mode pass through VBS₁ with the transmittance of t_1 to generate a mixed state in the a_3 . If the SPS is reflected to the a_4 mode, it means that the photon is lost. The photon in the mode b_1 can be used to judge the single-photon in a_1 mode. By changing t_1 , we can obtain the different mixed state. The photon in both a_2 and b_2 mode are used as the auxiliary photons. After making it pass through VBS₂ with the transmittance $t_2 < \frac{1}{2}$, we make the photons in the a_3 and b_4 modes pass through the BS₁ and detect the photon in the d_1 and d_2 modes. In this way, the first level of amplification can be realized. Next, the photon in the a_2 mode is used as the second auxiliary photon. Similarly, we make it pass through VBS₃ with the transmittance $t_2 < \frac{1}{2}$, and then make the photons in the a_6 and b_3 modes pass through the BS₂. We can finally realize the second amplification. In current technology, with the help of cascaded SPDC sources, the generation of eight-photon entanglement has been realized [48, 49]. Such cascaded SPDC sources can be used to implement the experiment for multi-level cascaded NLA protocol.

- [20] S. Kocsis, G. Y. Xiang, T. C. Ralph, and G. J. Pryde, *Nat. Phys.* **9**, 23 (2013).
- [21] E. Meyer-Scott, M. Bula, K. Bartkiewicz, A. Černoč, J. Soubusta, T. Jennewein, and K. Lemr, *Phys. Rev. A* **88**, 012327 (2013).
- [22] K. Bartkiewicz, A. Černoč, K. Lemr, J. Soubusta, and M. Stobińska, *Phys. Rev. A* **89**, 062322 (2014).
- [23] N. Gisin, S. Pironio, and N. Sangouard, *Phys. Rev. Lett.* **105**, 070501 (2010).
- [24] M. Curty, and T. Moroder, *Phys. Rev. A* **84**, 010304(R) (2011).
- [25] S. L. Zhang, S. Yang, X. B. Zou, B. S. Shi, and G. C. Guo, *Phys. Rev. A* **86**, 034302 (2012).
- [26] L. Zhou, and Y. B. Sheng, *J. Opt. Soc. Am. B* **30**, 2737 (2013).
- [27] Y. B. Sheng, Y. Ou-Yang, L. Zhou, and L. Wang, *Quant. Inf. Process.* **13**, 1595 (2014).
- [28] T. J. Wang, C. Cao and C. Wang, *Phys. Rev. A* **89**, 052303 (2014).
- [29] D. Pitkanen, X. Ma, R. Wickert, P. van Loock, and N. Lütkenhaus, *Phys. Rev. A* **84**, 022325 (2011).
- [30] G. Y. Xiang, T. C. Ralph, A. P. Lund, N. Walk, and G. J. Pryde, *Nat. Photon.* **4**, 316 (2010).
- [31] N. A. McMahon, A. P. Lund, and T. C. Ralph, *Phys. Rev. A* **89**, 023846 (2014).
- [32] C. Silberhorn, T. C. Ralph, N. Lütkenhaus, and G. Leuchs, *Phys. Rev. Lett.* **89**, 167901 (2002).
- [33] C. Silberhorn, N. Korolkova, and G. Leuchs, *Phys. Rev. Lett.* **88**, 167902 (2002).
- [34] M. G. A. Paris, M. Cola, and R. Bonifacio, *Phys. Rev. A* **67**, 042104 (2003).
- [35] G. M. D'Ariano and P. Lo Presti, *Phys. Rev. Lett.* **86**, 4195 (2001).
- [36] G. M. D'Ariano, P. Lo Presti, and M. G. A. Paris, *Phys. Rev. Lett.* **87**, 270404 (2001).
- [37] N. Sangouard, C. Simon, T. Coudreau, and N. Gisin, *Phys. Rev. A* **78**, 050301(R) (2008).
- [38] D. Salart, N. Sangouard, N. Gisin, H. Herrmann, B. Sanguinetti, C. Simon, W. Sohler, and R.T. Thew, *Phys. Rev. Lett.* **104**, 180504 (2010).
- [39] Y. B. Sheng, F. G. Deng, and H. Y. Zhou, *Quantum Inf. Comput.* **10**, 272 (2010).
- [40] L. Zhou, and Y. B. Sheng, *Opt. Commun.* **313**, 217 (2014).
- [41] L. Zhou, Y. B. Sheng, W. W. Cheng, L. Y. Gong, and S. M. Zhao, *J. Opt. Soc. Am. B* **30**, 71 (2013).
- [42] L. Heaney, A. Cabello, M.F. Santos, and V. Vedral, *New J. Phys.* **13**, 053054 (2011).
- [43] D. Gottesman, T. Jennewein, and S. Croke, *Phys. Rev. Lett.* **109**, 070503 (2012).
- [44] C. Simon and J. W. Pan, *Phys. Rev. Lett.* **89**, 257901 (2002).
- [45] J. W. Pan, S. Gasparoni, R. Ursin, G. Weihs, and A. Zeilinger, *Nature* **423**, 417 (2003).
- [46] Y. B. Sheng, F. G. Deng, and H. Y. Zhou, *Phys. Rev. A* **77**, 042308 (2008).
- [47] Y. B. Sheng, and F. G. Deng, *Phys. Rev. A* **81**, 032307 (2010).
- [48] Y. F. Huang, B. H. Liu, L. Peng, Y. H. Li, L. Li, C. F. Li, and G. C. Guo, *Nat. Commun.* **2**, 546 (2011).
- [49] X. C. Yao, T. X. Wang, P. Xu, H. Lu, G. S. Pan, X. H. Bao, C. Z. Peng, C. Y. Lu, Y. A. Chen, and J. W. Pan, *Nat. Photon.* **6**, 225 (2012).